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Model independent constraints from vacuum and in-medium QCD Sum Rules

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Abstract. We discuss QCD sum rule constraints based on moments of vector meson spectral distributions in the vacuum and in a nuclear medium. Sum rules for the two lowest moments of these spectral distributions do not suffer from uncertainties related to QCD condensates of dimension higher than four. We exemplify these relations for the case of the ω meson and discuss the issue of in-medium mass shifts from this viewpoint.

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QCD sum rules have repeatedly been used in recent times to arrive at estimates for possible in-medium mass shifts of vector mesons [1,7]. The validity of such estimates has been questioned, however, for several reasons. First, for broad structures such as the ρ meson whose large vacuum decay width is further magnified by in-medium reactions, the QCD sum rule analysis does not provide a reliable framework to extract anything like a "mass shift" [3,5]. Secondly, notorious uncertainties exist at the level of factorization assumptions commonly used to approximate four-quark condensates in terms of $\langle\bar{q}q\rangle^2$, the square of the standard chiral condensate. The first objection is far less serious for the ω meson which may well have a chance to survive as a reasonably narrow quasi-particle in nuclear matter [3,4]. The second objection, however, is difficult to overcome: factorization of four-quark condensates may indeed be questionable.

In the present note we focus on the two lowest moments ($\int ds s^n R(s)$ with $n = 0, 1$) of vector meson spectral distributions, in vacuum as well as in nuclear matter, and point out that these are subject to sum rules which do

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not suffer from the uncertainties introduced by four-quark condensates. These sum rules are shown to provide useful, model independent constraints which we exemplify for the case of the ω meson spectral distribution and its change in the nuclear medium. The sum rule for the second moment, $\int ds s^2 R(s)$, does involve the four-quark condensate. In fact it can be used in principle to determine this particular condensate and test the factorization assumption. The detailed analysis of this question will be defered to a longer paper. In this short note we confine ourselves to conclusions that can be drawn without reference to four-quark condensates.

The starting point is the current-current correlation function

$$\Pi_{\mu\nu}(q) = i \int d^4x \, e^{iq\cdot x} \langle \mathcal{T} j_\mu(x) j_\nu(0) \rangle \tag{1}$$

where $\mathcal{T}$ denotes the time-ordered product and the expectation value is taken either in the vacuum or in the ground state of nuclear matter at rest. In vacuum the polarization tensor (1) can be reduced to a single scalar correlation function, $\Pi(q^2) = \frac{1}{3} g^{\mu\nu} \Pi_{\mu\nu}(q)$. In nuclear matter there are two (longitudinal and transverse) correlation functions which coincide for a meson at rest with respect to the medium (i.e. with $q^\mu = (\omega, \mathbf{q} = 0)$).

The reduced correlation function is written as a (twice subtracted) dispersion relation,

$$\Pi(q^2) = \Pi(0) + \Pi'(0)\, q^2 + \frac{q^4}{\pi} \int ds \frac{\mathrm{Im}\Pi(s)}{s^2(s - q^2 - i\epsilon)}. \tag{2}$$

where $\Pi(0)$ vanishes in vacuum but contributes in nuclear matter. At large spacelike $Q^2 = -q^2 > 0$ the QCD operator product (Wilson) expansion gives

$$12\pi^2 \Pi(q^2 = -Q^2) = -c_0 Q^2 \ln\left(\frac{Q^2}{\mu^2}\right) + c_1 + \frac{c_2}{Q^2} + \frac{c_3}{Q^4} + \ldots \tag{3}$$

We specify the coefficients c_i for the isoscalar current $j^\mu = \frac{1}{6}(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d)$, the case of the ω meson that we wish to use here for explicit evaluations. In vacuum we have:

$$c_0 = \frac{1}{6}\left(1 + \frac{\alpha_S}{\pi}\right), \qquad c_1 = -\frac{1}{2}(m_u^2 + m_d^2), \tag{4}$$

$$c_2 = \frac{\pi^2}{18} \langle \frac{\alpha_S}{\pi} \mathcal{G}^{\mu\nu} \mathcal{G}_{\mu\nu} \rangle + \frac{2\pi^2}{3} \langle m_u u\bar{u} + m_d d\bar{d} \rangle, \tag{5}$$

while c_3 involves combinations of four-quark condensates of (mass) dimension 6. The quark mass term c_1 is small and can be dropped in the actual calculations. For the gluon condensate we use $\langle \frac{\alpha_S}{\pi} \mathcal{G}^2 \rangle = (0.36\,\mathrm{GeV})^4$ [9], and the (chiral) quark condensate is given by $\langle m_u \bar{u}u + m_d \bar{d}d \rangle \simeq m_q \langle \bar{u}u + \bar{d}d \rangle = -m_\pi^2 f_\pi^2 \simeq -(0.11\,\mathrm{GeV})^4$ through the Gell-Mann, Oakes, Renner relation.

In the nuclear medium with baryon density ρ we have $c_i(\rho) = c_i(\rho = 0) + \delta c_i(\rho)$ with $c_i(0)$ given by eqs.(4,5), and

$$\delta c_2(\rho) = \frac{\pi^2}{3} \left[-\frac{4}{27} M_N^{(0)} + 2\sigma_N + A_1 M_N \right] \rho \tag{6}$$

to linear order in ρ. The first term in brackets is the leading density dependent correction to the gluon condensate and involves the nucleon mass in the chiral limit, $M_N^{(0)} \simeq 0.75\,\mathrm{GeV}$ [6]. The second part proportional to the nucleon sigma term $\sigma_N \simeq 45\,\mathrm{MeV}$ is the first order correction of the quark condensate, and the third term introduces the first moment of the quark distribution function in the nucleon:

$$A_1 = 2 \int dx\, x \, \left[u(x) + \bar{u}(x) + d(x) + \bar{d}(x) \right]. \tag{7}$$

It represents twice the fraction of momentum carried by quarks in the proton. We take $A_1 \simeq 1$ as determined by deep-inelastic lepton scattering at $Q \sim 2\,\mathrm{GeV}$. Note that $\delta c_2(\rho_0) \simeq 4{\cdot}10^{-3}\,\mathrm{GeV}^4$ at $\rho = \rho_0 = 0.17\,\mathrm{fm}^{-3}$, the density of nuclear matter, and almost all of this correction comes from the term proportional to A_1.

Next we introduce the Borel transform of eq. (3):

$$12\pi^2 \Pi(0) + \int_0^\infty ds\, R(s) e^{-s/\mathcal{M}^2} = c_0 \mathcal{M}^2 + c_1 + \frac{c_2}{\mathcal{M}^2} + \frac{c_3}{2\mathcal{M}^4} + \dots \tag{8}$$

with $R(s) = -\frac{12\pi}{s}\mathrm{Im}\Pi(s)$ and $\Pi(0) = -\rho/4M_N$, the vector meson analogue of the Thomson term in photon scattering.

We separate the spectrum $R(s)$ into a resonance part with $s \le s_0$ and a continuum $R_c(s)$ which must approach the perturbative QCD limit for $s > s_0$:

$$R_c(s) = \frac{1}{6}\left(1 + \frac{\alpha_S}{\pi}\right) \Theta(s - s_o). \tag{9}$$

The factor $\frac{1}{6}$ is again specific for the isoscalar channel. The Borel mass parameter $\mathcal{M}$ must be sufficiently large so that eq.(8) converges rapidly, but otherwise it is arbitrary. We choose $\mathcal{M} > \sqrt{s_0}$ so that $e^{-s/\mathcal{M}^2}$ can be expanded in powers of $s/\mathcal{M}^2$ for $s < s_0$. The remaining integral $\int_{s_0}^\infty ds\, R_c(s) e^{-s/\mathcal{M}^2}$ is evaluated inserting the running coupling strength $\alpha_S(s_o)$ in eq.(9). Then the term-by-term comparison in eq.(8) gives the following set of sum rules for the moments of the spectrum $R(s)$ (see also refs. [7, 8])

$$\int_0^{s_0} ds\, R(s) = s_0 c_0 + c_1 - 12\pi^2 \Pi(0), \tag{10}$$

$$\int_0^{s_0} ds\, s\, R(s) = \frac{s_0^2}{2} c_0 - c_2, \tag{11}$$

$$\int_0^{s_0} ds\, s^2\, R(s) = \frac{s_0^3}{3} c_0 + c_3. \tag{12}$$

Note that the first two sum rules are well determined and represent useful constraints for the spectrum $R(s)$. Only the third sum rule (12) involves four-quark condensates which are uncertain. In this short paper we concentrate on eqs. (10,11). A detailed analysis of eq.(12) will be presented in a forthcoming longer paper. It is instructive to illustrate the sum rules (10,11) for the ω meson in vacuum using Vector Meson Dominance (VMD) for the resonant part of $R(s)$. In this model we have

$$R(s) = 12\pi^2 \frac{m_\omega^2}{g_\omega^2}\, \delta(s - m_\omega^2) + \frac{1}{6}\left(1 + \frac{\alpha_S}{\pi}\right) \Theta(s - s_o). \tag{13}$$

with $g_\omega = 3g \simeq 16.8$ (using the vector coupling constant $g = 5.6$). We can neglect the small quark mass term c_1 and find from eq. (10):

$$\frac{8\pi^2}{g^2} \frac{m_\omega^2}{s_0} = 1 + \frac{\alpha_S}{\pi}, \tag{14}$$

which fixes $\sqrt{s_0} = 1.16\,\mathrm{GeV}$ using $\alpha_S(s_0) \simeq 0.4$ and $m_\omega = 0.78\,\mathrm{GeV}$. It is interesting to identify the spectral gap $\Delta = \sqrt{s_0}$ with the scale for spontaneous chiral symmetry breaking, $\Delta = 4\pi f_\pi$, where $f_\pi = 92.4\ MeV$ is the pion decay constant. In the VMD model, taking the zero width limit, eq.(14) holds for both the ω and ρ meson, with equal mass $m_V = m_\rho = m_\omega$. Inserting $s_0 = 16\pi^2 f_\pi^2$ in eq.(14) one recovers the famous KSFR relation $m_V = \sqrt{2}\, g f_\pi$ up to a small QCD correction.

The sum rule (11) for the first moment gives

$$\frac{8\pi^2}{g^2} m_\omega^4 = \frac{s_0^2}{2}\left(1 + \frac{\alpha_S}{\pi}\right) - \frac{\pi^2}{3}\left[\langle \frac{\alpha_S}{\pi} \mathcal{G}^2 \rangle + 12 \langle m_u \bar{u} u + m_d \bar{d} d \rangle\right]. \tag{15}$$

Inserting the values for the gluon and quark condensates we find indeed perfect consistency. Given a model for the ω meson spectral function in the vacuum and in the nuclear medium, the sum rules (10,11) therefore provide useful constraints to test the calculated spectra.

We now continue on from VMD to a more realistic approach. In refs. [3,4] we have used an effective Lagrangian based on chiral $SU(3) \otimes SU(3)$ symmetry with inclusion of vector mesons as well as anomalous couplings from the Wess-Zumino action in order to calculate the ω meson spectrum both in the vacuum and in nuclear matter. The resulting vacuum spectrum reproduces the observed $e^+e^- \to$ hadrons$(I=0)$ data very well [3] (see Fig. 1a). The predicted in-medium mass spectrum (for ω excitations with $\mathbf{q}=0$) shows a pronounced downward shift of the ω-meson peak and a substantial, but not overwhelming increase of its width from reactions such as $\omega N \to \pi N$, $\pi\pi N$ etc. (see Fig. 1b). At large $s > s_0$, both spectra should approach the QCD limit (9). The consistency test of these calculated spectral distributions with the sum rules (10) and (11) goes as follows:

– <u>the vacuum case</u>:

the two sides of eq. (10),

$$\int_0^{s_0} ds\, R(s) = \frac{s_0}{6}\left(1+\frac{\alpha_S(s_0)}{\pi}\right), \tag{16}$$

now match at $\sqrt{s_0} = 1.25\,\mathrm{GeV}$, with $\int_0^{s_0} ds\, R(s) = 0.29\,\mathrm{GeV}^2$. The sum rule for the first moment gives $\int_0^{s_0} ds\, s\, R(s) = 0.19\,\mathrm{GeV}^4$, to be compared with $\frac{1}{2}s_0^2(1+\alpha_s/\pi) - c_2 = 0.22\,\mathrm{GeV}^4$, so there is consistency at the 10% level.

– <u>the in-medium case</u>:

now we have to match the moments of the density dependent spectral distributions,

$$\int_0^{s_0} ds\, R(s,\rho) = \frac{s_0}{6}\left(1+\frac{\alpha_S(s_0)}{\pi}\right) + \frac{3\pi^2}{M_N}\rho, \tag{17}$$

together with

$$\int_0^{s_0} ds\, s\, R(s,\rho) = \frac{s_0^2}{12}\left(1+\frac{\alpha_S(s_0)}{\pi}\right) - c_2(0) - \delta c_2(\rho). \tag{18}$$

Using our calculated spectrum [4] shown in Fig. 1b, we find $\sqrt{s_0} = 1.08\,\mathrm{GeV}$ at $\rho = \rho_0 = 0.17\,\mathrm{fm}^{-3}$, with $\int_0^{s_0} ds\, R(s,\rho_0) = 0.26\,\mathrm{GeV}^2$. Then $\int_0^{s_0} ds\, s\, R(s,\rho_0) = 0.11\,\mathrm{GeV}^4$ is to be compared with the right hand side of eq. (18) which gives $0.12\,\mathrm{GeV}^4$, so there is again excellent consistency.

Note again that these tests do not involve uncertain four-quark condensates. Furthermore, if the in-medium spectrum shows a reasonably narrow quasi particle excitation, the quantity $\bar{m}^2 = \int_0^{s_0} ds\, s\, R(s) / \int_0^{s_0} ds\, R(s)$ can indeed be interpreted as the square of an in-medium "mass" of this excitation. For our ω meson case we find $\bar{m} = 0.65\,\mathrm{GeV}$ at $\rho = \rho_0$, a substantial downward mass shift as discussed in refs. [3,4]. (For the broad ρ meson spectrum, on the other hand , the interpretation of $\bar{m}$ as an in-medium mass is not meaningful as demonstrated in ref. [3]).

Amusingly, the spectral gap $\Delta = \sqrt{s_0}$ decreases by about 15 percent when replacing the vacuum by nuclear matter. This is in line with the proposition that this gap reflects the order parameter for spontaneous chiral symmetry breaking and scales like the pion de-

cay constant f_π (or, equivalently, like the square root of the chiral condensate $\langle \bar{q}q \rangle$).

In summary, we have shown that the combination of sum rules (10) and (11) for the lowest moments of the spectral distributions does serve as a model-independent consistency test for calculated spectral functions.

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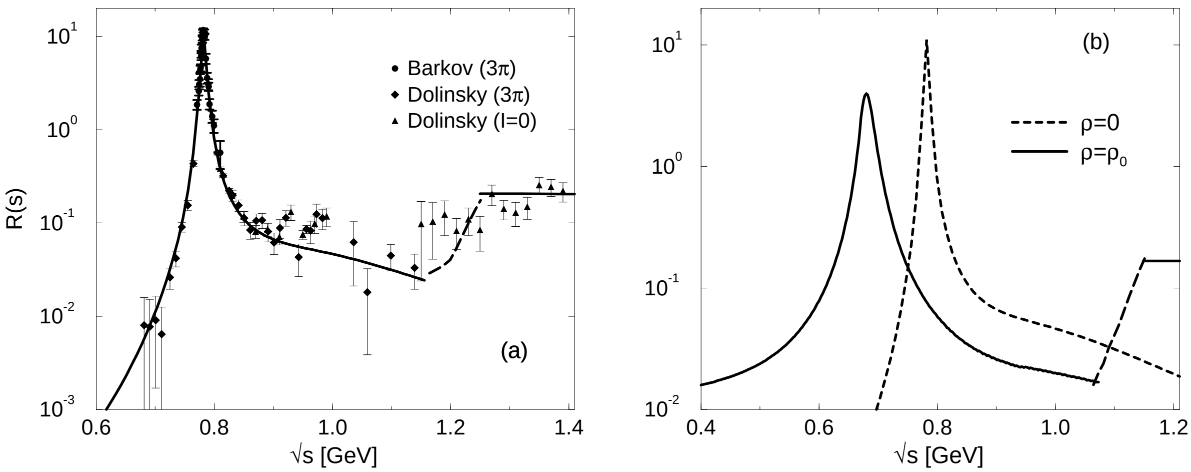

Fig. 1. a) Spectrum $R(s)$ in the ω meson channel as calculated in ref. [3] (solid line). The data points refer to $e^+e^- \to 3\pi$ and $e^+e^- \to$ hadrons $(I = 0)$ [10]

b) In-medium spectrum of ω meson excitations in nuclear matter at density $\rho_0 = 0.17\,\mathrm{fm}^{-3}$ as calculated in refs.[3,4] (solid line) in comparison with the vacuum spectrum (dashed line).